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A dynamics toolbox for modelling and prediction of interconnected systems

Wouter Jutte[1], Nard Strijbosch[1], Max Bolderman[1], Teunis van Dam[1], Arjan van der Wel[1], Ruben Merks[1], Theresa Spaan-Burke[1], Henny Spaan[1]

[1]*IBS Precision Engineering BV*

jutte@ibspe.com

Abstract

Developing new and innovative industrial machines requires delicate balancing of cutting-edge requirements, tight deadlines and limited resources in an environment with many uncertainties. Early identification of issues and quickly adapting is the recipe for success. This requires flexible and efficient analysis tools for continuous monitoring of the expected system performance as the design progresses. In high-performance mechatronic systems the dynamic behaviour is often of critical importance. IBS Precision Engineering BV developed a dynamics toolbox for dynamic system analysis, which changes the analysis from a complex coding exercise to a simple bookkeeping one. This enables quick design iteration cycles with continuous model refinements, a continuous overview of system performance, and allows the analysis to be performed by employees with limited background in the mathematics of dynamics. The dynamics toolbox uses a modular approach often used in systems engineering. This paper describes the concepts and workflow of the dynamics toolbox. The dynamics toolbox is applied to a use-case of an adjustable mirror and the model is verified with experimental measurements.

V-model, FEA, dynamics, toolbox, rigid body modelling, FEA, system identification

1. Introduction

IBS Precision Engineering BV is a leading supplier in the precision industry for new, innovative and custom industrial machines and applications. In these applications the dynamic performance is often of critical importance. One example is a project for laser satellite communication, where optical systems have to aim laser beams over very long distances. The mirrors in these systems have to be stable in the presence of vibrations or else the beam will miss the target as illustrated in Figure 1.

At IBS, the product creation process of new systems follows the V-model illustrated in Figure 2, where user requirements are converted into detailed designs using a top-down approach on the left side of the V-model and performance is verified on the right side using a bottom-up approach. To identify issues early, the system performance is modelled in parallel with the design process, where the models progress in detail as the project does.

The modelling approach from a dynamic perspective is shown on the left side of the V-model in Figure 2. Early concepts and specifications are determined with the use of first order approximations and 1D rigid body models with estimated masses, stiffnesses and forces based on practical experience. As the concepts are worked out, system design is refined and the models are updated with more realistic values. Once the geometry and interface locations become clear the rigid body models are upgraded with inertial and stiffness parameters in 3D. As the design matures, the complex and critical components are analysed with Finite Element Analysis (FEA) software and rigid bodies can be replaced with exported Finite Element Method (FEM) models with many degrees of freedom (DoF).

The process of creating and updating models can be very time consuming. Large mass, damping and stiffness matrices have to be assembled for complex 3D systems with many components. Doing so in a programming environment like MATLAB is a cumbersome task and requires a significant amount of knowledge about the mathematics of dynamics. FEA software is a very powerful tool for detailed analysis of structural dynamics. However, it doesn't lend itself well for first order estimations in the early phases of the product creation process. Also, analysing large complex systems with many components in FEA software is very computationally expensive, which is not effective for frequent design iterations.

Because the available tools did not accommodate our needs well enough, a dynamics toolbox is developed with the goal of making dynamic modelling easy and efficient throughout the whole product creation process.

The remainder of this work is organized as follows: Section 2 describes the concepts of the dynamics toolbox, section 3 applies the modelling approach to a mirror adjustment module, in section 4 the model is experimentally verified, section 5 discusses the results and section 6 concludes the work.

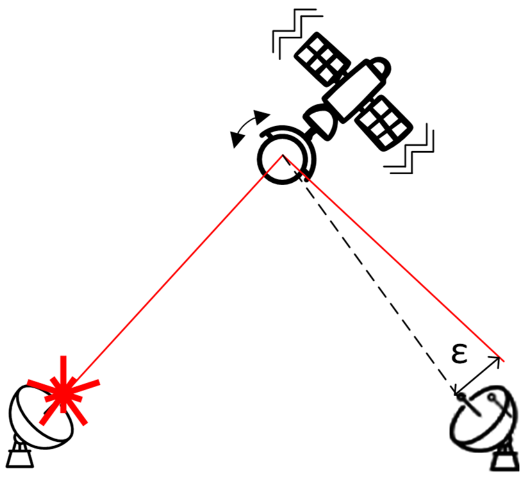

Figure 1. Schematic of beam deflection as a result of a vibrating mirror in Laser satellite communication

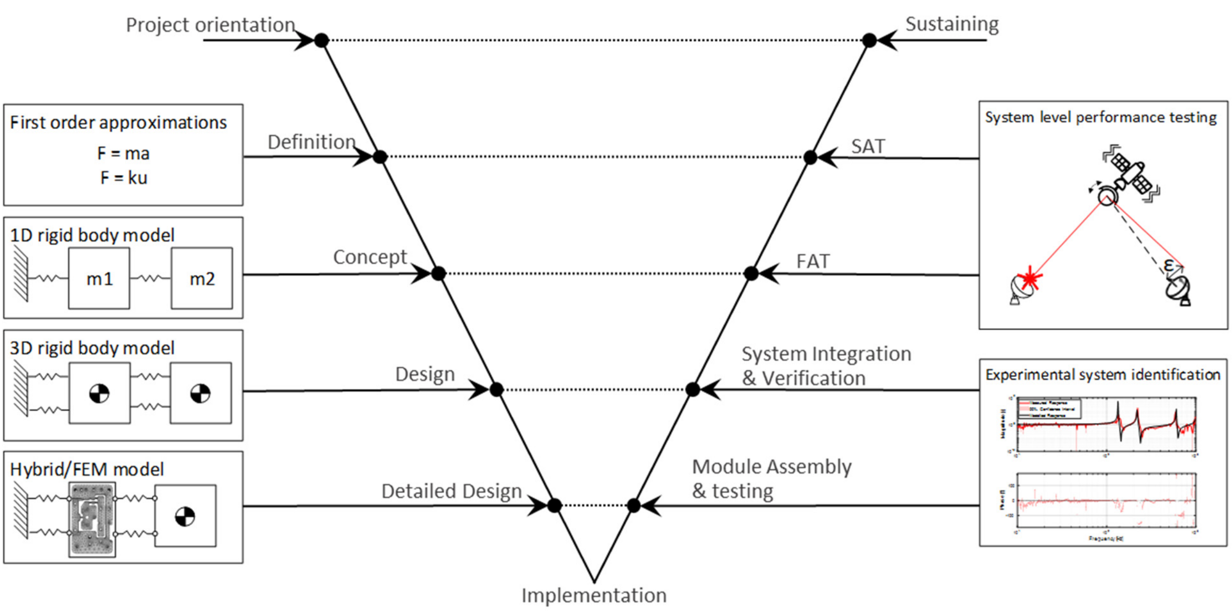

Figure 2. The V-model from the dynamic's perspective. On the left side of the V-model system performance estimations are done based on models, which progress in parallel with designs from first order approximations to assembled FEM models. In the early phases of system integration on the right side of the V-model dynamics are tested by making frequency response functions. In the last phases, system level performance is tested as a whole.

2. Dynamics toolbox concepts

The core functionality of the dynamics toolbox is to reduce the modelling process to the specification of components and their parameters. The different components and parameters used in the toolbox are shown in Table 1. With the specification, the dynamic toolbox automatically interconnects the components and constructs the system matrices of the standard matrix equation (1) which forms the basis for dynamic analysis:

$$\boldsymbol{M}\ddot{x} + \boldsymbol{C}\dot{x} + \boldsymbol{K}x = F, \quad (1)$$

where $\boldsymbol{M}$ is the inertial matrix**,** $\boldsymbol{C}$ the damping matrix**,** $\boldsymbol{K}$ the stiffness matrix, F the force vector, x the displacement vector, $\dot{x}$ the velocity vector and $\ddot{x}$ the acceleration vector.

2.1. System Matrix Assembly

The usefulness of the automation concept is illustrated with the 1D rigid body system in shown Figure 3 (a). The first step in the manual system matrix assembly procedure is to draw a free-body-diagram. The generic free-body-diagram for each body in 1D is shown in Figure 3 (b). Using this generic free-body-diagram, the equations of motion for the bodies can be written down:

$$m_1\ddot{x}_1 = f_1 - k_1x_1 - c_1\dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1), (2)$$

$$m_2\ddot{x}_1 = -k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1)\,, \quad (3)$$

where (2) is the equation of motion for the first rigid body and (3) is the equation of motion for the second rigid body. The system of equations can be rewritten in the standard matrix form (1) with:

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \qquad F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$
$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \qquad \boldsymbol{C} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}$$
$$\ddot{x} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The complexity of this modelling process increases significantly when adding more bodies and extending the model to 3D. In 3D, 6 equations of motion have to be created for each rigid body (3 translations and 3 rotations). Forces and torques also have a direction and no longer necessarily act in the centre of gravity (CoG), which complicates modelling further.

To reduce the effort, the process of the system assembly procedure is automated. The dynamics toolbox uses an approach similar to the generic free-body-diagram in Figure 3 (b), but then in 3D with the component parameters of Table 1.

Table 1 The components and parameters for 3D system specification in the dynamics toolbox.

Component	**Parameters**
Fixation	3D position of centre of rotation (x, y, z)
Rigid body	3D CoG position (x, y, z) Inertial parameters (m, I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz})
Flexible body	Mass, damping and stiffness matrix (exported from FEA software)
Spring/ damper connection	3D position of interface on body 1 (x, y, z) 3D position of interface on body 2 (x, y, z) Stiffness parameters (k_x, k_y, k_z, k_{xx}, k_{yy}, k_{zz}) Relative damping ratio (ζ)

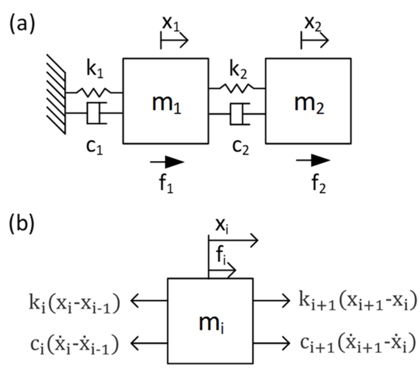

Figure 3. (a) A 1D model with 2 rigid bodies. (b) The generic free-body-diagram for a rigid body in 1D.

2.2. Interfacing with FEM models

In addition to automated system matrix assembly of rigid body models, the dynamics toolbox accommodates interfacing with flexible bodies, which are models exported from FEA software. FEA software is used to create discretized models with many DoF. In the dynamics toolbox, spring/damper connections can be specified between rigid bodies and interfaces of the exported models. It is also possible to connect multiple exported models.

To reduce the computational cost, model-order-reduction (MOR) is applied to the exported FEM models. MOR reduces the DoFs from about 10^5 - 10^6 to 10 - 100 for a typical industrial component. However, some level of accuracy is lost with MOR, which is heavily influenced by the method used. Component mode synthesis [2] is most commonly used in structural dynamics.

3. Modelling of a mirror adjustment module

To showcase how the dynamics toolbox is used in practice, the next section applies the dynamics toolbox to the development process of a mirror adjustment module.

As already described in the introduction, the orientation and stability of mirrors in optical systems is of critical importance. Installation on mechanical tolerances is not always sufficient to align the optical components. Therefore, adjustable mirror mounts are commonly used for fine alignment.

The flexible connection introduced to adjust the mirror can cause problems in the presence of vibrations. The stiffness of the fine adjustment mechanism should be flexible enough to make the adjustment possible, but stiff enough so that it doesn't oscillate too much with respect to the frame it is fixated on.

3.1. Module requirements

The maximum allowable mirror deflection δ (rad) is determined in the initial stages of the V-model, which follows from the application's maximum allowed beam deflection ε, as illustrated in Figure 1. To start the modelling and design process, these specifications have to be converted to specifications for stiffness and inertia.

To get an initial approximation for stiffness and inertia, the module is considered as a 1D single body rigid body model. For rotation around the Z-axis we can use the following equations:

$$T = I_{zz}\ddot{\varphi}_z, \qquad (4)$$
$$T = k_{zz}\delta, \qquad (5)$$

where T (Nm) is the Torque, I_{zz} (kg·m^2) the moment of inertia, k_{zz} (Nm/rad) the rotational stiffness and $\ddot{\varphi}_z$ (rad/s^2) the rotational acceleration. Substitution of (4) in (5) and reordering gives a stiffness requirement [1]:

$$k_{zz} \geq I_{zz}\sqrt{\frac{\ddot{\varphi}_z}{\delta}}. \qquad (6)$$

Given the maximum allowed mirror deflection δ, the expected accelerations $\ddot{\varphi}_z$ exerted on the module and an approximation of I_{zz} based on the material and an estimation of the geometry, the stiffness requirement follows.

Instead of considering the adjustable mirror as a single module, it can be decomposed in two well-defined sub-modules: the adjustment flexure and the mirror. Such a decomposition is useful, because it allows the sub-modules to be independently designed, modelled and iterated on. The approximated stiffness and moment of inertia are distributed over the two sub-modules.

3.2. Concept phase: 1D rigid body model

After an initial specification of the moments of inertia and stiffnesses, 1D rigid body models are typically used for modelling of concept designs. Modelling of the adjustable mirror with 2 sub-modules gives the 1D rigid body model illustrated in top left of the V-model in Figure 2, where the flexure is connected to the fixation and the mirror is connected to the flexure.

Five components have to be defined to model this 1D rigid body system in the dynamics toolbox: The fixation, 2 rigid bodies and 2 spring/damper connections.

For 1D less parameters are required than in Table 1 for 3D. The location of the bodies and the location of the connection interfaces don't matter in 1D. 1D also requires only one stiffness parameter per connection and one inertial term per body. The relative damping is optional. The system can therefore be specified with:

- The inertial parameters for the bodies $I_{zz,mirror}$ and $I_{zz,flexure}$.
- The stiffness parameters $k_{zz,flexure}$ *and* $k_{zz,mirror}$ for the connections fixation-flexure and flexure-mirror respectively.

These parameters are enough to assemble the system matrices and start 1D system analysis.

3.3. Design phase: 3D rigid body models.

As the project advances in the V-model from the concept phase to the design phase, the geometry and the locations of the interfaces become clearer. In this phase the rigid body models can be upgraded to 3D as illustrated in Figure 2. For 3D analysis all parameters in Table 1 have to be specified.

The inertial parameters, interface locations and stiffnesses can be extracted from commonly used CAD software.

3.4. Detailed design phase: Hybrid/FEM models.

As the designs mature into detailed designs, the final geometry of components becomes clear. The designs of the sub-modules for the adjustable mirror are shown in Figure 4.

The flexure dynamics are complex and a simple rigid body approximation is not good enough to represent the real flexure dynamics. In such a case, the rigid body can be replaced by an exported FEM model. The CAD model is imported in FEA software, where the meshing functionality is used to discretize the model and the created system matrices are then exported.

In the dynamics toolbox connections are defined between the fixation and interfaces of the flexure's FEM model, and between interfaces of the FEM model and the rigid body of the mirror. This hybrid model is illustrated in the bottom left of the V-model in Figure 2.

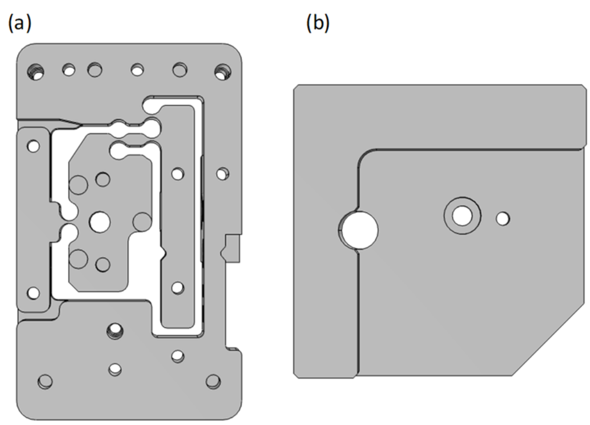

Figure 4. The mechanical designs of (a) the adjustment flexure (b) the mirror.

4. Experimental verification of a mirror adjustment module

After the designs are finished the parts are ordered, manufactured and delivered and the right side of the V-model is started. On the right side of the V-model the system is integrated and verified using a bottom-up approach. The dynamic verification of (sub-)modules is done experimentally by means of system identification, as shown on the right side of the V-model in Figure 2.

4.1. Background

The goal of dynamic system identification is to find the Frequency Response Function (FRF) that maps movement at the input to movement at the output as a function of frequency.

Figure 5 illustrates how the FRF is experimentally obtained for a 1D system. The system is excited at the input with sinusoidal signals of different frequencies. While doing the excitation, the displacement profile at both the input and the output are measured. For each excitation frequency the measured signal at the input and output have the same frequency, but the input and output have a different amplitude and phase. The FRF contains this magnitude and phase relationship between input and output for all the tested frequencies.

The system identification process is very similar in 3D, but the inputs and outputs are measured in 6-DoF and 6 linearly independent excitations have to be applied, 1 excitation for each DoF.

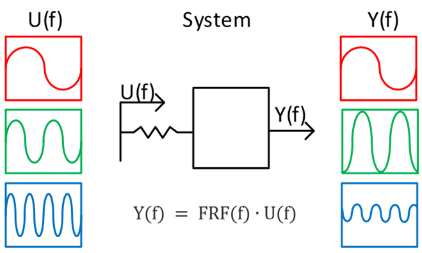

Figure 5. A schematic of system identification in 1D. Input U is excited while measuring input U and output Y to determine FRF(f).

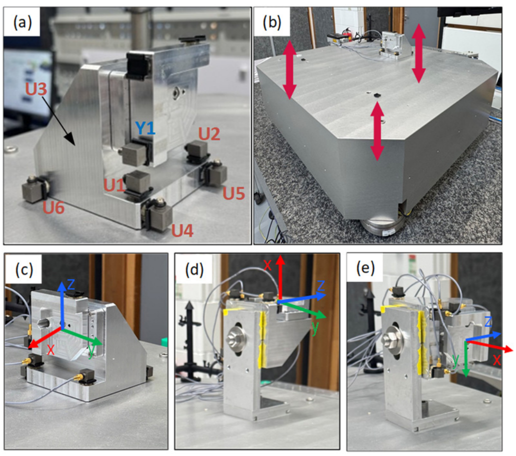

Figure 6. The experimental setup. (a) Shows the location of the 6 input accelerometers U1-U6 and the output accelerometer Y1. (b) The excitation platform has 3 piezos, which can excite in Z/Rx/Ry. (c, d, e) The module is rotated into 3 different orientations, so that all 6 degrees of freedom can be excited.

4.2. Experiment

The experimental setup for the verification of the mirror adjustment module is shown in Figure 6. Figure 6 (a) shows the placement of the accelerometers. To measure all 6-DoF at the input, six accelerometers are placed on a bracket, which is connected to the input of the flexure. Only one accelerometer is placed on the mirror. It is not necessary to measure all 6-DoF of the output at the same time, but it would be more time efficient. To get the full 6-DoF system identification with only one accelerometer, the experiments have to be repeated 6 times while moving the output accelerometer to different positions.

The module is connected to an excitation platform with 3 piezo actuators, shown in Figure 6 (b). With three actuators, the system can only be excited in 3-DoF, i.e., in z, Rx and Ry, see Figure 6 (b). To excite the system in all 6-DoF, the experiment is repeated for different orientations, see Figure 6 (c, d, e). In all orientations a multi-sine excitation is applied to the platform across a range of frequencies while logging all accelerometers.

An FRF is obtained that maps the movement of each measured input to the movement of each measured output. The spectral analysis method [3] is used to get FRFs from the experiments, including a confidence interval of the experimental result.

5. Results

Figure 7 shows the comparison between the experimental results and the model for one of the obtained frequency responses: from input U6 to output Y1 in Figure 6 (a). The modelled response is shown as the dashed black line and the experimental result as a solid red line. The experimental result includes a 99% confidence interval, which shows some uncertainty in the lower frequencies of the measured FRF. This aligns with our expectations, since the used accelerometers are noisier for low frequencies. Apart from that, the modelled and experimental response are very similar.

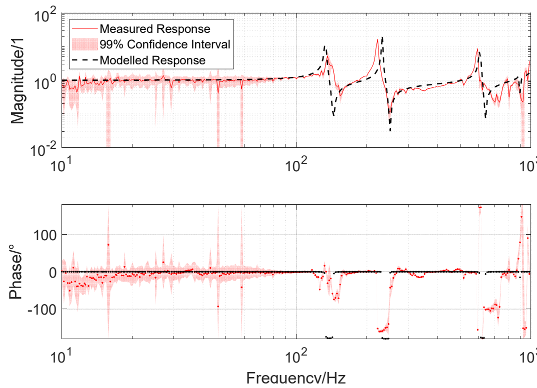

Figure 7. A comparison of the modelled and the experimentally obtained FRF for Y1/U6. Confidence intervals are plotted for the experimental FRF, which show the bounds of the real FRF with 99% confidence.

6. Conclusion

A dynamics toolbox is developed that can automated the assembly procedure of system matrices for dynamic system analysis and it also accommodates interfacing with FEM models. To get the system matrices, parameters of dynamic objects are specified instead of manually doing the assembly procedure. This makes modelling easier and faster. The toolbox allows for incremental upgrades, which accommodate the product creations process of the V-model. The dynamics toolbox is applied to model a mirror adjustment module and the theoretical response is verified experimentally.

References

[1] Munnig Schmidt R, Schitter G, Rankers A and van Eijk J 2020, *The Design of High Performance Mechatronics* (Amsterdam: IOS Press) p128

[2] Hurty W C 1960 *Vibrations of structural systems by component mode synthesis*. J. of the Eng. Mech. Division, 86(4):51–69

[3] Pintelon R and Schoukens J 2012, *System Identification* (Hoboken New Jersey: John Wiley & Sons, Inc.) pp 233-236